

On the cosmological vacuum and dark energy

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Abstract. – A degenerate fermionic vacuum population is suggested. Based on the abundance of the dark energy density in the Universe the vacuum particle mass and number density are estimated. The obtained mass is in reasonable agreement with observations and predictions of the neutrino mass. Number densities are higher than those of the relativistic relic neutrino component.

1 Introduction

By definition, the vacuum is the zero point of everything we measure: energy, temperature and so on. Recent astrophysical observations (Riess et al., 1998; Perlmutter et al., 1999a) tell convincingly that it is not empty. It contains a substantial energy density that is attributed to some unknown ingredient (Zeldovich, 1967, 1968; Turner, 1998; Perlmutter et al., 1999b) and its quantum fluctuations. This energy content is negative while playing the role of a cosmological constant Λ (Einstein, 1917; Dirac, 1937; Wilczek, 1984; Weinberg, 1989; Carroll et al., 1992) in a quasi-Einstein-de Sitter Universe.

Let us assume the vacuum had temperature $T = 0$ (in energy units), a temperature far away from any temperature at which any of the high energy quantum field theories would come to work, except of quantum electrodynamics. At zero temperature (or close to it, which in cosmological context means at times well after element generation, at least, however, after decoupling) the real vacuum in the cosmos is subject to electrodynamics only. It, for instance, implies that we can use electromagnetic waves as carrier of information and can measure everything against the stable electrodynamic vacuum. Another consequence is buried in the constancy of the electrodynamic properties of the vacuum, the constancy of its electric and magnetic susceptibilities μ_0 and ϵ_0 which includes the constancy of the resistance of the vacuum.

This contemporary electrodynamic vacuum, if containing a real (virtual) particle component (virtual particles of mass,

charge, spin) must be subject to the Fermi distribution. The only bosons propagating on it at our temperatures are photons and gravitons, the former constituting the electrodynamic fields. Moreover, since the vacuum is our zero-point reference, any particle component it contains cannot have positive energy. This had already been proposed by P.A.M. Dirac when discovering his relativistic equation of the electron. However, as it turned out and was interpreted differently in QED, neither electrons nor positrons, their anti-particles, don't populate the vacuum.

At negative energy levels $\epsilon_p = -\sqrt{\mathbf{p}^2 c^2 + m_v^2 c^4} = -\epsilon_{vp} < 0$ (or its Diracian equivalent) and low temperature $T_v \equiv \beta_v^{-1} < m_v c^2$, assumed to be less than the hypothetical particle rest mass energy $m_v c^2$ of the vacuum particles, the Fermi distribution (in the absence of negative temperatures¹) implies that the average occupation number of states is $\bar{n}_p = 1$. All negative energy states are occupied. Summed over all states from $-\infty$ to 0 would yield an infinite vacuum particle number, which cannot be true. The number of particles respectively their number density N_v should be finite also in the vacuum. Thus not all negative energy levels are occupied. Instead the negative fully populated sea is somehow mirror symmetric to the positive domain of particle energies at zero temperature. Inspection of the Fermi distribution for negative energies at $T_v = 0$

$$\bar{n}_p = \left[e^{-\beta_v(\epsilon_{vp} + \mu_v)} + 1 \right]^{-1} \quad (1)$$

with μ_v the chemical potential, suggests that the chemical potential in the vacuum must be negative, $\mu_v < 0$, if the Fermi distribution should make sense also there. Occupation of negative energy levels is in this case limited by the negative vacuum Fermi energy

$$\mu_v \equiv -\epsilon_{vF} = -\frac{\hbar^2}{2m_v} \left(\frac{6\pi^2}{g} N_v \right)^{\frac{2}{3}} \quad (2)$$

¹For the non-existence of negative temperatures $T < 0$ see (Dunkel and Hilbert, 2014). In fact, temperatures are defined from kinetic theory through mean square velocity differences (Δv^2) which, for positive occupation numbers of states, are always positive definite.

where N_v is the unknown and, in principle, not directly measurable vacuum particle density. Levels $\epsilon_p < \epsilon_{vF}$ are empty, while levels $\epsilon_{vF} \leq \epsilon_p \lesssim 0$ are occupied with $\bar{n}_p = 1$. Moreover by Dirac's original relativistic arguments energy levels above the negative rest mass energy $-m_v c^2$ are also empty in order to prevent leakage of particles from the vacuum sea to reality unless injection of radiative energy would lead to spontaneous particle pair creation out of the nothing, i.e. out of the vacuum. This requires that the vacuum particles could interact with radiation which, to present knowledge, seems not to be the case, except for the cosmological effect of the negative vacuum energy density.

2 Estimates

Little is known so far about the vacuum population. The above Fermi energy, a completely unknown quantity, contains two undetermined quantities, the vacuum particle mass, m_v , which for real particles is positive, and the vacuum particle number density N_v .

We can, however, impose conditions. The first is that the vacuum Fermi energy, for a sufficiently large number of negative energy states to exist in the vacuum, should at least equal the rest-mass energy $-\epsilon_{vF} \sim -m_v c^2$ or equivalently $\epsilon_{vF} \sim m_v c^2$ where ϵ_{vF} is taken positive. This gives

$$m_v c^2 \lesssim \frac{\hbar^2}{2m_v} \left(\frac{6\pi^2}{g} N_v \right)^{\frac{2}{3}} \quad (3)$$

Astronomical observations of the accelerated expansion of the universe (Riess et al., 1998; Perlmutter et al., 1999a) tell that our epoch is characterized by the increasing dominance of the vacuum pressure which drives the cosmological expansion. Our current cosmological epoch is special in the sense that vacuum and baryonic energy densities are of same order of magnitude. The vacuum energy density, however, is constant while the mass density decreases with cosmological expansion. Hence, it increasingly overcomes both the visible as well as dark matter contributions plus the cosmic background radiation pressure including any other radiative components. From these observations a negative vacuum energy density $-\rho_\Lambda$ has been inferred which seems to be either exactly or at least very close to the value expected for a stationary, i.e. time-independent vacuum. The index Λ relates the cosmological vacuum energy density to the cosmological constant Λ (Wilczek, 1984; Weinberg, 1989; Carroll et al., 1992). Writing for the vacuum energy density $N_v \epsilon_{vF}$ and setting it equal to the observed cosmological value we have

$$N_v \epsilon_{vF} \sim \rho_\Lambda \quad (4)$$

Since $\rho_\Lambda = \Omega_\Lambda \rho_{crit}$ is known, with ρ_{crit} the critical cosmological energy density, the two above equations provide a means of estimating both the number density N_v of particles in the cosmological vacuum and the mass of the hypothetical Fermionic population of the vacuum.

Define a relative vacuum particle mass $\varpi_v = m_v/m$, where m is the electron mass. Then, from the last equality follows for the vacuum number density

$$N_v = \left(\frac{2m\rho_\Lambda}{\hbar^2} \right)^{\frac{3}{2}} \left(\frac{g}{6\pi^2} \right)^{\frac{2}{3}} \varpi_v^{\frac{3}{2}} \quad (5)$$

Inserting this into the condition on the vacuum Fermi energy (4) yields an estimate for the relative mass of the vacuum Fermions

$$\varpi_v \lesssim \frac{\epsilon_{vF}(\varpi_v)}{mc^2} \sim \frac{\rho_\Lambda}{mc^2 N_v} \quad (6)$$

which is an implicit equation. Resolving it gives

$$\varpi_v \lesssim \left(\frac{3}{4} \frac{\pi^2}{g} \right)^{\frac{1}{4}} \left(\frac{\rho_\Lambda \lambda_C^3}{mc^2} \right)^{\frac{1}{4}} \approx 1.34 \left(\frac{\rho_{crit} \lambda_C^3}{mc^2} \right)^{\frac{1}{4}} \quad (7)$$

where the Compton wavelength of an electron $\lambda_C = \hbar/mc$ has been introduced. The central part of this expression depends only on the one free parameter Ω_Λ which is subject to determination making use of astronomical data. $\Omega_\Lambda \approx 0.7$ and $g = 1.75$ for Fermions gives the numerical factor. This is independent on the number density N_v . Hence, it can be used to obtain a limit on ϖ_v . Inserting into the above expression for the vacuum density it also permits obtaining an important independent vacuum density estimate

$$N_v \lesssim 1.1 \left(\frac{g}{6\pi^2} \right)^{\frac{1}{4}} \left(\frac{\rho_\Lambda}{mc^2 \lambda_C} \right)^{\frac{3}{4}} \approx 0.85 \left(\frac{\rho_{crit}}{mc^2 \lambda_C} \right)^{\frac{3}{4}} \quad (8)$$

which may be considered as an upper bound. The critical mass density in the Universe is $\rho_m = 9.21 \times 10^{-27} \text{ kg/m}^3$ which, in terms of the electron rest mass energy mc^2 , yields a critical energy density of $\rho_{crit}/mc^2 \approx 1.01 \times 10^4$. This number gives a rest mass

$$m_v = \varpi_v m_e \sim 5.05 \times 10^{-9} m_e \quad (9)$$

of the hypothetical particles that make up the vacuum population. Using the rest mass energy of an electron, this becomes

$$m_v \lesssim 2.6 \times 10^{-3} \text{ eV}/c^2 \quad (10)$$

a small number which indicates that the massive vacuum Fermions should be light. With this small mass the vacuum number density is found to be of the order

$$N_v \lesssim 1.75 \times 10^{12} \text{ m}^{-3} \quad (11)$$

Such a fermionic vacuum number density is surprisingly high when compared with the densities of interstellar, galactic or even intergalactic space. It is also four orders of magnitude larger than the relativistic relic neutrino background density $N_v^{rel} \approx 3.3 \times 10^8 \text{ m}^{-3}$ in the Universe. Such a vacuum is quite dense though being very dilute when compared with a vacuum composed of Planckian particles of mass $m_{Pl} \sim 10^{19} \text{ GeV}/c^2$.

Lower limits on both these numbers are currently not known theoretically. Considering the experimental inference on Ω_Λ we note that the ever more precise measurements have decreased its value slowly from $\Omega_\Lambda \gtrsim 0.7$ to its presently best value $\Omega_\Lambda \approx 0.695$. In order to be on the very conservative side we may thus boldly assume that, at least to current intelligence, an absolute lower observational limit will not be less than $\Omega_\Lambda = 0.6$. This gives a factor of 1.29 instead of 1.34 in (7), restricting the vacuum particle mass to the narrow range

$$2.5 \times 10^{-3} < m_\nu \lesssim 2.6 \times 10^{-3} \text{ eV}/c^2 \quad (12)$$

The vacuum number density of the hypothetical particles is, by the same reasoning, subject to the bounds

$$1.56 \times 10^{12} < N_\nu \lesssim 1.75 \times 10^{12} \text{ m}^{-3} \quad (13)$$

These densities are the famous disturbing ~ 110 orders of magnitudes below a hypothetical vacuum number density based on the assumption of a vacuum built of Planckions. In terms of mass ratios between Planckions of mass m_{Pl} and the vacuum particles this reduces to a “somewhat fairer” $m_{\text{Pl}}/m_\nu \sim 30$ orders of magnitude discrepancy. If the current theory has anything in common with reality then Planckions are ruled out by it at least for the contemporary vacuum as it is experienced over all the astronomical observational times in our past-inflation epoch.

3 The question for the particles

Not many Fermions this light are known to us. The most promising particles are neutrinos of which we know that they interact only weakly with matter. The combined current limits from cosmological probes (Cuesta et al., 2016) so far available are upper bounds $m_\nu = \sum_i m_{\nu_i} < 0.12 \text{ eV}/c^2$ which do not rule out that the real vacuum neutrino mass would be somewhat lower. Neutrino oscillations of real (i.e. non-vacuum) relativistic non-degenerate neutrinos indicate non-vanishing mass differences $|\Delta m_{\nu_i}|^2$ between the three different generations of neutrinos. In contrast to the free relativistic neutrino background and neutrinos produced in the Universe in β decay and other elementary interactions, vacuum neutrinos are degenerate at $T_\nu \approx 0$. Theory suggests that the masses of the three degenerate neutrino generations are of same order of magnitude $m_\nu \sim |\Delta m_{\nu_i}| \sim 5 \times 10^{-2} \text{ eV}/c^2$. Observations (on the 2σ confidence level) indicate $\Delta m_{\nu_i}^2 \gtrsim 8 \times 10^{-5} (\text{eV}/c^2)^2$ yielding $m_\nu \sim 10^{-2} \text{ eV}/c^2$ which is somewhat though not very much larger than our estimated vacuum particle mass range. When measurements will be pushed down to $< 0.1 \text{ eV}/c^2$, for instance by KATRIN (Mertens, 2016), this will soon be in the reach to become checked.

The theoretical normal hierarchical scheme currently predicts $m_\nu \lesssim 5 \times 10^{-3} \text{ eV}/c^2$ for the electron neutrino, which is very close to the above completely independent estimate (12).

The usual notion that the vacuum is not empty but hosts quantum fluctuations, i.e. virtual particle-antiparticle pairs,

would suggest that the degenerate vacuum consist of pairs of virtual neutrinos which are their own antiparticles. Such neutrinos satisfy Majorana statistics instead of Dirac statistics and are known as Majorana neutrinos. Current degenerate experimental upper bounds for Majorana neutrino masses are $m_\nu \lesssim 0.1 \text{ eV}/c^2$, which is again not in disagreement with the above independent estimate. The current discrepancy of less than two orders of magnitude is surprisingly small.

Given the independent assumptions made in our estimates, it is hard to believe that this closeness respectively approximate similarity of the numbers would just be a coincidence. One may thus become inclined to identify the vacuum mass with the lightest (say electron) neutrino mass

$$m_\nu \approx m_{\nu_e} \quad (14)$$

of degenerate neutrinos populating the vacuum being in negative energy states. For a continuum of vacuum energy levels the neutrinos form a degenerate Landau-Fermi fluid (Landau et al., 1980). Such a fluid is subject to quantum fluctuations, which at the low vacuum temperatures propagate as low intensity zero sound at sound speed $c \geq u_0 \gtrsim p_{\text{vF}}/m_\nu$. Since $p_{\text{vF}} \sim m_\nu c$, one concludes that

$$u_0 = c \quad (15)$$

Wavenumbers $k_0 < m_\nu c/\hbar$ imply wavelengths $\lambda_0 \gtrsim 18 \text{ Mpc}$, which is larger than $> 0.1\%$ of the radius of the Universe and exceeds the diameter of all galaxy clusters. On shorter scales this vacuum can be considered as completely homogeneous. Thus vacuum fluctuations would be interpreted as a spectrum of large-scale zero sound fluctuations propagating at the velocity of light. Longest “visible wavelengths” would be of the order of the diameter of the visible Universe.

The vacuum energy density has not changed over the entire time of existence of the vacuum as we know it after the end of inflation. For the same reason the zero-sound speed has been constant over this period. Its constancy implies the constancy of the velocity of light as a property of the invariable vacuum.

Inverting the argument one may conjecture that the velocity of light is a constant of nature because it is a constant property of the degenerate vacuum which has been invariant for most of the lifetime of the Universe.

4 Cosmological consequences

The visible Universe contains a non-degenerate relativistic dilute neutrino population of cosmological very-early universal origin and current temperature $T_\nu/k_B \approx 1.9 \text{ K}$. These neutrinos have decoupled at roughly $t \sim 1 \text{ s}$ after the Big Bang. Though being of finite mass they by current intelligence probably do not contribute much to the unknown composition of Dark Matter because of their low density and relativistic speed close to the velocity of light. It is certain that they do not participate in the Dark Energy.

The hypothetical degenerate non-relativistic vacuum neutrinos must also have been produced in the very early Universe, presumably before or at least during the inflation phase as a component of the then existing “hot vacuum” which is assumed to have been unstable with respect to the phase transition that led to onset of inflation. During inflation they cooled down to the new low, and from our positive energy point of view negative, degenerate energy state filling the vast domain of the Universe up to its inflation-extended size before settling at a time around some $t \sim 10^{-32}$ s or so into its current quasi-stationary or even time-independent state. They form an about homogeneous background in the Universe, the vacuum as we experience it, extending far out over the horizons of the visible Universe. It is then no surprise that the vacuum energy density does not vary, at least over those times and distances which have been accessible to observations. This is in accord with Dirac’s original hypothesis (Dirac, 1937) that Λ is about constant in our present cosmological epoch. This epoch is just post-inflationary long.

The interesting question concerns the origin of the vacuum neutrino component. Speculatively they might have been produced prior to or during inflation as the lightest Fermionic decay products of the Planckions, the smallest known (or imaginable) Black Holes, which constituted the pre-inflationary vacuum in case these evaporated when the Universe started inflating. It is believed that their interior is subject to quantum gravity.

Black Holes have finite temperature, behave like black bodies, and emit Hawking radiation of virtual particle pairs. The horizon of Planckions is the Planck radius ℓ_{Pl} , with uncertainty $\Delta\ell_{\text{Pl}} \sim \ell_{\text{Pl}}$ being of the same order as the radius itself. Thus Planckions appear to the outside as diffuse specks of Planck density which suggests that their interior is in the chromo-plasma state of free quarks and gluons. If not in thermal equilibrium with their vacuum environment, in particular when the vacuum is empty, they irradiate within Planck-Hawking times $\tau_{\text{H}} \lesssim 10^4 \tau_{\text{Pl}} \sim 10^{-40}$ s, i.e. rather soon after generation in the Big Bang.

Hawking radiation of Black Holes represent local anti-gravity, a well-known fact. For Planckions it overcomes gravity and lets them evaporate. One may ask which mass would just balance the collapse. This happens when the Hawking time $\tau_{\text{H}} \approx \tau_{\text{coll}}$ equals the collapse time. For a Schwarzschild-Black Hole $2\tau_{\text{coll}}/\pi = \sqrt{3/8\pi G\rho(R_s)}$ has been given for a dust cloud (Oppenheimer and Snyder, 1939; Weinberg, 1972) consisting of particles that interact only gravitationally, with collapse starting at Schwarzschild-radius R_s , with density $\rho(R_s)$ and zero free-fall velocity. Equating both expressions² yields for the collapse-balancing mass $M_{\text{cb}} \approx \sqrt{\pi} m_{\text{Pl}}$, which is slightly larger than the Planck mass. Smaller masses, including the Planck mass, if confined to their Schwarzschild radii, will evaporate completely before col-

lapse. Masses larger than the Planck mass have shorter internal collapse times. Hawking radiation cannot stop them from collapsing.

The lifetime of Planckions will in addition depend on the primordial vacuum conditions, the vacuum temperature, respectively the inflaton field Φ . In thermal equilibrium with their environment, Planckions will be stable because the same amount of radiation they lose by Hawking radiation they absorb in return from the environment. In the other case all Planckions should decay, but possibly only slowly, if the thermal equilibrium is only weakly violated. This may have been the case when the inflaton was in its original meta-stable state. One may expect that, when the pre-inflationary vacuum temperature approached the order of the electroweak transition at about $\sim 10^{-36}$ s, neutrinos and other particles (massive Bosons) might have become radiated away into the surrounding vacuum as virtual particles.

The time difference between Planck (or Planck-Hawking evaporation) time and onset of inflation around $\sim 10^{-34} - 10^{-33}$ s, believed to be of the order of few orders of magnitude in time, should provide information about the physical state (temperature, density, composition) of the pre-inflationary vacuum and prolongation of the decay time of Planckions in the very early Universe.

What enabled the vacuum in the course of inflation to trap the majority of neutrinos, which in the above scenario might have been produced prior to inflation, letting them disappear our direct inference, is not known. From their comparably large density one may suspect that they represent the bulk component of the pre-inflationary neutrino population, with the free cosmological relic neutrino background just forming the inflationary run-away tail of the pre-inflation neutrino distribution that escaped trapping by the inflating vacuum. Constituting the vacuum component the degenerate negative-energy vacuum neutrinos (or virtual neutrino pairs), being trapped by the post-inflationary vacuum, were not affected by and thus did not participate in the subsequent reheating of matter.

One might wonder whether today the vacuum neutrinos, in addition to providing the Dark Energy density, could contribute to the unknown composition of Dark Matter. This is not the case, however, as can be shown by asking up to what radial distance R from a large local mass $M = \alpha M_{\odot}$, with M_{\odot} the solar mass, the local gravitational energy density and thus gravitational attraction could overcome the Dark Energy pressure of the light degenerate vacuum neutrinos. This leads to

$$R < \frac{GM_{\odot} m N_{\nu}}{\Omega_{\Lambda} \rho_m c^2} \alpha \varpi_{\nu} \approx 10^{-9} \alpha \text{ m} \quad (16)$$

Even the largest clusters of galaxies hosting $\alpha \sim 10^{14}$ solar masses, if concentrated within one point, would dominate the Dark Energy merely within a sphere of ~ 100 km radius from their centers. A light neutrino vacuum can thus barely

²This is allowed because both times refer to their common zero point of time at the Schwarzschild horizon.

be made responsible for the Dark Matter content of the Universe.

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